Simplexová metóda: Cvičenie 6 a 7

Príklad 1

min 3x1 + 2x2 - x3 - 2x4 - 5x5 + 6x6

st 5x2 + 3x4 + 3x5 + x6 = 20  
 x1 + 4x2 + x4 + 2x5 = 24  
 - 2x2 + x3 + x4 + 7x5 = 17  
 xj ≥ 0, j=1..6

Riešenie

**Vznik novej podmienky:**  
 x0 = 3x1 + 2x2 – x3 – 2x4 – 5x5 + 6x6/ *- x0*  
 x0 – 3x1 – 2x2 + x3 + 2x4 + 5x5 – 6x6 = 0  
  
 min x0  
 st x0 - 3x1 - 2x2 + x3 + 2x4 + 5x5 - 6x6 = 0  
 5x2 - 3x4 + 3x5 + x6 = 20  
 x1 + 4x2 + x4 + 2x5 = 24  
 - 2x2 +x3 + x4 + 7x5 = 17  
 xj ≥ 0, j=1..6  
**Redukovaný kánonický tvar:**  
Poznámka: stĺpce x1, x2 a x3 tvoria bázu, a preto sa musia vynulovať, čiže:  
 - **x1** => [x1] + 4x2 + x4 + 2x5 = 24/ *- 4x2 – x1 – 2x5*,  
 x1 = 24 – 4x2 – x4 – 2x5.  
 - **x3** => - 2x2 + [x3] + x4 + 7x5 = 17/ *+ 2x2 – x4 – 7x5*,  
 x3 = 17 + 2x2 – x4 – 7x5.  
 - **x6** => 5x2 – 3x4 + 3x5 + [x6] = 20/ *- 5x2 + 3x4 – 3x5*,  
 x6 = 20 – 5x2 + 3x4 – 3x5.  
  
 min x0  
 st x0 + 42x2 + 22x4 + 22x5 = 175  
 5x2 + 3x4 + 3x5 + x6 = 20  
 x1 + 4x2 + x4 + 2x5 = 24  
 - 2x2 + x3 + x4 + 7x5 = 17  
 xj ≥ 0, j=1..6, x0 ∈ R  
Poznámka: každá maximalizačná úloha sa dá prerobiť do minimalizačnej úlohy.  
**Tabuľky:**  
Stĺpcové pravidlo (výber pivota): pre vstup do bázy vyberieme nebázický stĺpec, ktorého redukovaná cena je najmenšia záporná (**najväčšia hodnota** – lebo sa pohybujeme v záporne vzatých redukovaných cenách!).  
 Tabuľka 0

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza | | x0 | x1 | x2 | x3 | x4 | x5 | x6 | b |  |
| 0 |  | 1 |  | *42* |  | 22 | 22 |  | 175 | > záporne vzaté redukované ceny (riadok x0) |
| 6 |  |  |  | **5** |  | 3 | 3 | 1 | 20 | 4 = 20 / 4 |
| 1 |  |  | 1 | *4* |  | 1 | 2 |  | 24 | 6 = 24 / 4 |
| 3 |  |  |  | *-2* | 1 | 1 | 1 |  | 17 | > pivot nesmie byť záporný! |

Hodnoty 42 a 22 v riadku x0 sú záporne vzaté redukované ceny, preto sú v skutočnosti záporné.  
Hľadáme najväčší pokles: 175 – 42 \* \_, ~~175 – 22 \* \_~~.  
 x = <24, 0, 17, 0, 0, 20> f(x) = 175

Poznámka: ak sú všetky redukované ceny nezáporné, už nie je možné získať žiadne riešenie s menšou hodnotou účelovej funkcie.  
 Tabuľka 1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza | | x0 | x1 | x2 | x3 | x4 | x5 | x6 | b |  |
| 0 |  | 1 |  |  |  | -16/5 | -16/5 | -42/5 | 7 | Táto tabuľka je už **optimálna**. |
| 2 |  |  |  | 1 |  | 3/5 | 3/5 | 1/5 | 4 |  |
| 1 |  |  | 1 |  |  | -7/5 | -2/5 | -4/5 | 8 |  |
| 3 |  |  |  |  | 1 | 11/5 | 41/5 | 2/5 | 25 |  |

**xopt = <8, 4, 25, 0, 0, 0>** f(x) = 7

Príklad 2

max 6x1 + 2x2  
 st 3x1 + x2 ≤ 12  
 x2 ≤ 3  
 xj ≥ 0, j = 1, 2

Riešenie

**Kánonický tvar:** min -6x1 - 2x2  
 st 3x1 + x2 + x3 = 12 // x3 a x4 sa nenachádza v prvom riadku  
 x2 + x4 = 3 // podmienok, preto ich nie je potrebné počítať.  
 xj ≥ 0, j = 1, ..., 4  
**Redukovaný kanonický tvar:**  
 min x0  
 st x0 + 6x1 + 2x2 = 0  
 3x1 + x2 + x3 = 12  
 x2 + x4 = 3  
 xj ≥ 0, j = 1, ..., 4  
 Tabuľka 0

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza | | x0 | x1 | x2 | x3 | x4 | b |  |
| 0 |  | 1 | *6* | 2 |  |  |  |  |
| 3 |  |  | **3** | 1 | 1 |  | 12 | 4 = 12 / 3 |
| 4 |  |  |  | 1 |  | 1 | 3 | > pivot nesmie byť 0! |

Tabuľka 1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Báza | | x0 | x1 | x2 | x3 | x4 | b |
| 0 |  | 1 |  |  | -2 |  | -24 |
| 1 |  |  | 1 | 1/3 | 1/3 |  | 4 |
| 4 |  |  |  | 1 |  | 1 | 3 |

**x1opt = <4, 0, 0, 3>** f(x) = 24  
Vysvetlenie: **-24** je záporne vzatá redukovaná cena a jedná sa o max úlohu, tak v skutočnosti je hodnota **+24**.  
 Podúloha (nepovinné)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Báza | | x0 | x1 | x2 | x3 | x4 | b |
| 0 |  | 1 |  |  | -2 |  | -24 |
| 1 |  |  | 1 |  | 1/3 | -1/3 | 3 |
| 2 |  |  |  | 1 |  | 1 | 3 |

**x2opt = <4, 0, 0, 3>** f(x) = 24

**Všetky optimálne riešenia:**  
α\*<4, 0, 0, 3>T + (1 – α)\*<3, 3, 0, 0>T = < α + 3, -3α + 3, 0, 3α >T, pre α ∈ <0, 1>

Príklad 3

min 4x1 -5x2

st x1 + x2 ≥ 10

x1 +2x2 ≤ 4

xj ≥ 0, j = 1, 2

Riešenie

**Kánonický tvar:**  
 min x0 = 4x1 - 5x2  
 st x1 + x2 - x3 = 10 // **mínus** kvôli opačnej nerovnosti  
 x1 + 2x2 + x4 = 4  
 xj ≥ 0, j = 1, ..., 4  
**Doplnenie umelých premenných (náhradná úloha):**  
 min x0 = x5  
 x1 + x2 - x3 + x5 = 10  
 x1 + 2x2 + x4 = 4  
 xj ≥ 0, j = 1, ..., 5  
Poznámka: ak náhradná úloha bude mať riešenie, ale pôvodná nie, tak {} prípustných riešení je prázdna.  
 x0 = x5  
 x0 = 10 – x1 – x2 + x3  
 x0 + x1 + x2 – x3 = 10  
**Redukovaný kánonický tvar náhradnej úlohy:**  
 min x0  
 st x0 + x1 + x2 - x3 = 10  
 x1 + x2 - x3 + x5 = 10  
 x1 + 2x2 + x4 = 4  
 xj ≥ 0, j = 1, ..., 5, x0 ∈ R  
 Tabuľka 0

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza | | x0 | x1 | x2 | x3 | x4 | x5 | b |  |
| 0 |  | 1 | *1* | 1 | -1 |  |  | 10 |  |
| 5 |  |  | *1* | 1 | -1 |  | 1 | 10 | 10 = 10 / 1 |
| 4 |  |  | **1** | 2 |  | 1 |  | 4 | 4 = 4 / 1 |

Klesne o: x1: -1\*4/1 => 10 – 4 = 6  
 x2: -1\*4/2 => 10 – 2 = 8  
 Tabuľka 1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza | | x0 | x1 | x2 | x3 | x4 | x5 | b |
| 0 |  | 1 |  | -1 | -1 | -1 |  | 6 |
| 5 |  |  |  | -1 | -1 | -1 | 1 | 6 |
| 1 |  |  | 1 | 2 |  | 1 |  | 4 |

**xopt = <4, 0, 0, 0, 6>** f(x) = 6 - optimálne riešenie náhradnej úlohy z umelou bázou.

Príklad 4

min -4x1 -7x2  
 st x1 + x2 ≥ 10  
 4x1 -5x2 ≤ 20  
 x1, x2 ≥ 0

Riešenie

**Kánonický tvar:**  
 min x0 = -4x1 - 7x2  
 st x1 + x2 - x3 +x5 ≥ 10  
 4x1 - 5x2 +x4 ≤ 20  
 xj ≥ 0, j = 1, ..., 5  
**Náhradná úloha:**  
 min x5  
 st x1 + x2 - x3 + x5 = 10  
 4x1 - 5x2 + x4 = 20  
 xj ≥ 0, j = 1, ..., 5  
 Tabuľka 0

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza |  | x0 | x1 | x2 | x3 | x4 | x5 | b |  |
| 0 |  | 1 | 1 | *1* | -1 |  |  | 10 | x1: (-1) \* 20/4 = -5 |
| 5 |  |  | 1 | **1** | -1 |  | 1 | 10 | x2: (-1) \* 10/1 = -10 |
| 4 |  |  | 4 | *-5* |  | 1 |  | 20 | najrýchlejší pokles |

Tabuľka 1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza |  | x0 | x1 | x2 | x3 | x4 | x5 | b |
| 0 |  | 1 |  |  |  |  | -1 |  |
| 2 |  |  | 1 | 1 | -1 |  | 1 | 10 |
| 4 |  |  | 9 |  | -5 | 1 |  | 70 |

Vyšla pôvodná úloha  
 Tabuľka 1 (riešenie 2)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Báza |  | x0 | x1 | x2 | x3 | x4 | b |
| 0 |  | 1 | -3 |  | 7 |  | -70 |
| 2 |  |  | 1 | 1 | -1 |  | 10 |
| 4 |  |  | 9 |  | -5 | 1 | 70 |

Tabuľka nie je optimálna a nie je možná vybrať pivota.  
 Úloha nemá riešenie z dôvodu neohraničenosti množiny prípustných riešení.  
**Určenie polpriamky:**

Do bázy malo ísť x3, ale nemožno vybrať pivota, preto je koniec.  
Úloha nemá riešenie z dôvodu neohraničenosti {} množiny prípustných riešení.  
 Polpriamka <0, 10, 0, 70> + α\*< 0, 1, 1, 5> α ≥ 0

Príklad 5

max 3x1 + x2 + 5x3 + 4x4 + x5  
 st x1 + 3x2 + 2x3 + x5 = 12  
 3x1 + x2 + x3 = 6  
 5x1 - x2 -2x3 + x4 = 10  
 x1, x2, x3, x4, x5 >= 0

Riešenie

min -3x1 - x2 - 5x3 - 4x4 -x5  
 st x1 + 3x2 + 2x3 + x5 = 12  
 3x1 + x2 + x3 + x6 = 6  
 5x1 - x2 - 2x3 + **x4** = 10  
 Vyznačené x4 si s ničím neprekáža, čiže nie je potrebná ďalšia premenná.  
**Náhradná úloha v redukovanom tvare:**  
 min x0  
 st x0 + 3x1 - x2 + x3 = 6  
 x1 + 3x2 + 2x3 +x5 = 12  
 3x1 + x2 + x3 + x6 = 6  
 5x1 - x2 - 2x3 + x4 = 10  
 Tabuľka 0

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza |  | x0 | x1 | x2 | x3 | x4 | x5 | x6 | x7 | b |
| 0 |  | 1 | 3 | 1 | 1 | 1 |  |  |  | 6 |
| 5 |  |  | 1 | 3 | 2 | 2 |  | 1 |  | 12 |
| 6 |  |  | 3 | 1 | 1 | 1 |  |  | 1 | 6 |
| 4 |  |  | 5 | -1 | -1 | -2 | 1 |  |  | 10 |

Degenerované riešenie  
 Tabuľka 1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza |  | x0 | x1 | x2 | x3 | x4 | x5 | x6 | x7 | b |
| 0 |  | 1 |  |  |  |  | -1 |  |  |  |
| 5 |  |  |  | 8/3 | 5/3 |  | 1 | 1 | -1/3 | 10 |
| 1 |  |  | 1 | 1/3 | 1/3 |  |  |  | 1/3 | 2 |
| 4 |  |  |  | -8/3 | -11/3 | 1 |  |  | -5/3 |  |

Dosadíme  
 x0 + 3x1 + x2 + 5x3 + 4x4 + x5 = 0  
 x0 + 3 (2 + x2 /3 – x3/ 3) + x2 + 5x3 + 4 (8x2/3 + 11x3/3) + (10 – 8x2/3 – 5x3/3) = 0  
 x0 + 8x2 + 17x3 = -16  
Tabuľka 2

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza |  | x0 | x1 | x2 | x3 | x4 | x5 | b |  |
| 0 |  | 1 |  | 8 | 17 |  |  | -16 |  |
| 5 |  |  |  | 8/3 | 5/3 |  | 1 | 10 | 6 |
| 1 |  |  | 1 | 1/3 | 1/3 |  |  | 2 | 6 |
| 4 |  |  |  | -8/3 | -11/3 | 1 |  |  |  |

**x = <2, 0, 0, 0, 10, 0>** f(x) = -16

Optimálna tabuľka:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Báza |  | x0 | x1 | x2 | x3 | x4 | x5 | b |
| 0 |  | 1 |  | -96/5 |  |  | -51/5 | -118 |
| 3 |  |  |  | 8/5 | 1 |  | 1 | 6 |
| 1 |  |  | 1 | -1/5 |  |  |  |  |
| 4 |  |  |  | 8/5 |  | 1 | 2 | 22 |

**x = <0, 0, 6, 22, 0>** f(x) = -118  
 Optimálne riešenie pôvodnej úlohy  
 **x = <0, 0, 6, 22, 0>** f(x) = 118